Compendium of plane and spherical trigonometry

1 Plane trigonometry

Properties and definitions

$\sin(-\alpha) = -\sin(\alpha)$ | $\cos(-\alpha) = \cos(\alpha)$
$\sin(90° - \alpha) = \cos(\alpha)$ | $\cos(90° - \alpha) = \sin(\alpha)$
$\sin(90°+\alpha) = \cos(\alpha)$ | $\cos(90°+\alpha) = -\sin(\alpha)$
$\tan(\alpha) = \sin(\alpha)/\cos(\alpha)$ | $\cot(\alpha) = \cos(\alpha)/\sin(\alpha)$
$\text{versine}(\alpha) = 1-\cos(\alpha) = 2\sin^2(\frac{\alpha}{2})$ | $\text{haversin}(\alpha) = \frac{1}{2}\text{versine}(\alpha) = \sin^2(\frac{\alpha}{2})$

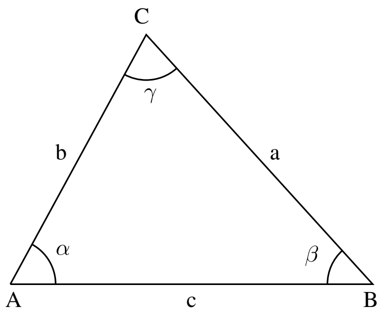

Figure 1: Triangle. The sides a,b and c are opposite to the angles α, β en γ.

Useful relations

$$\sin(\alpha) + \sin(\beta) = 2\sin[\tfrac{1}{2}(\alpha+\beta)]\cos[\tfrac{1}{2}(\alpha-\beta)] \quad (1.1)$$
$$\sin(\alpha) - \sin(\beta) = 2\cos[\tfrac{1}{2}(\alpha+\beta)]\sin(\tfrac{1}{2}(\alpha-\beta)] \quad (1.2)$$
$$\cos(\alpha) + \cos(\beta) = 2\cos[\tfrac{1}{2}(\alpha+\beta)]\cos[\tfrac{1}{2}(\alpha-\beta)] \quad (1.3)$$
$$\cos(\alpha) - \cos(\beta) = -2\sin[\tfrac{1}{2}(\alpha+\beta)]\sin[\tfrac{1}{2}(\alpha-\beta)] \quad (1.4)$$

$$2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta) \quad (1.5)$$
$$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta) - \sin(\alpha-\beta) \quad (1.6)$$
$$2\cos(\alpha)\cos(\beta) = \cos(\alpha+\beta) + \cos(\alpha-\beta) \quad (1.7)$$
$$2\sin(\alpha)\sin(\beta) = -\cos(\alpha+\beta) + \cos(\alpha-\beta) \quad (1.8)$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \tag{1.9}$$
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta) \tag{1.10}$$
$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta) \tag{1.11}$$
$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta) \tag{1.12}$$

Cosine rule

$$a^2 = b^2+c^2-2bc\cos(\alpha) \tag{1.13}$$
$$b^2 = a^2+c^2-2ac\cos(\beta) \tag{1.14}$$
$$c^2 = a^2+b^2-2ab\cos(\gamma) \tag{1.15}$$

Sine rule

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c} \tag{1.16}$$

2 Vectors and angles between them

In a Cartesian frame $\mathbf{\hat{u}_x}$, $\mathbf{\hat{u}_y}$ en $\mathbf{\hat{u}_z}$ will represent vectors of unit length along the x-, y- and z- axis respectively. A vector $\vec{\mathbf{a}}$, having projections a_x, a_y en a_z on the three axes reads:

$$\vec{\mathbf{a}} = a_x\mathbf{\hat{u}_x} + a_y\mathbf{\hat{u}_y} + a_z\mathbf{\hat{u}_z} \tag{2.1}$$

and its length is:

$$a = \mid \vec{\mathbf{a}} \mid = \sqrt{a_x^2+a_y^2+a_z^2} \tag{2.2}$$

Similarly, another vector, $\vec{\mathbf{b}}$, may be written as:

$$\vec{\mathbf{b}} = b_x\mathbf{\hat{u}_x} + b_y\mathbf{\hat{u}_y} + b_z\mathbf{\hat{u}_z} \tag{2.3}$$

with length:

$$b = \mid \vec{\mathbf{b}} \mid = \sqrt{b_x^2+b_y^2+b_z^2} \tag{2.4}$$

The angle θ between the two vectors is found from their scalar product $\vec{\mathbf{a}}.\vec{\mathbf{b}}$:

$$\vec{\mathbf{a}}.\vec{\mathbf{b}} = ab\cos(\theta) = a_xb_x + a_yb_y + a_zb_z \tag{2.5}$$

When the lengths of both a and b are already $= 1$, then $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are unit vectors themselves and we may denote them as $\mathbf{\hat{a}}$ and $\mathbf{\hat{b}}$. Then, equation 2.5 gives:

$$\cos(\theta) = \mathbf{\hat{a}}.\mathbf{\hat{b}} = a_xb_x + a_yb_y + a_zb_z \tag{2.6}$$

The scalar product of $\mathbf{\hat{a}}$ with itself gives the square of its length:

$$\mathbf{\hat{a}}.\mathbf{\hat{a}} = a_x^2+a_y^2+a_z^2 = 1 \tag{2.7}$$

Likewise for $\mathbf{\hat{b}}$.

3 Spherical coördinates. Cosine rule and sine rule

We define a Cartesian frame with axes (x,y,z). See figure 1. Along these three axes we introduce unit vectors $\hat{\mathbf{u}}_\mathbf{x}$, $\hat{\mathbf{u}}_\mathbf{y}$ and $\hat{\mathbf{u}}_\mathbf{z}$. We also give the direction of a unit vector $\hat{\mathbf{r}}$. Its spherical coördinates are the polar angle, θ, enclosed between the z-axis and $\hat{\mathbf{r}}$ and the azimuthal angle, ϕ, between the projection of $\hat{\mathbf{r}}$ onto the x-y-plane and the x-axis.

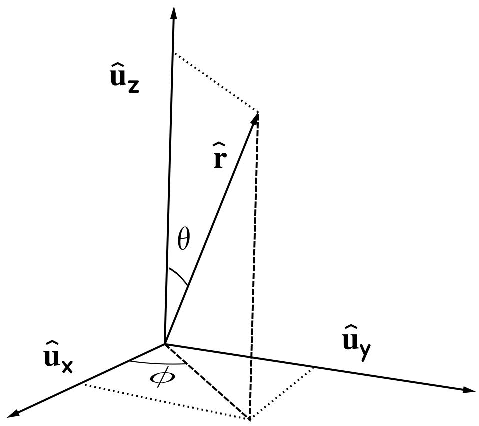

Figure 2: Connection between Cartesian and spherical coördinates.

It may be seen from the figure that in Cartesian coördinates the unit vector $\hat{\mathbf{r}}$ reads:

$$\hat{\mathbf{r}} = \hat{\mathbf{u}}_\mathbf{x}\sin(\theta)\cos(\phi) + \hat{\mathbf{u}}_\mathbf{y}\sin(\theta)\sin(\phi) + \hat{\mathbf{u}}_\mathbf{z}\cos(\theta) \tag{3.1}$$

For two unit vectors, $\hat{\mathbf{r}}_1(\theta_1, \phi_1)$ and $\hat{\mathbf{r}}_2$ (θ_2, ϕ_2), the angle Θ between them is found from their scalar product $\hat{\mathbf{r}}_1.\hat{\mathbf{r}}_2$:

$$\begin{aligned}\cos(\Theta) &= \sin(\theta_1)\cos(\phi_1)\sin(\theta_2)\cos(\phi_2) + \sin(\theta_1)\sin(\phi_1)\sin(\theta_2)\sin(\phi_2) + \cos(\theta_1)\cos(\theta_2) \\ &= \sin(\theta_1)\sin(\theta_2)[\cos(\phi_1)\cos(\phi_2) + \sin(\phi_1)\sin(\phi_2)] + \cos(\theta_1)\cos(\theta_2) \\ &= \sin(\theta_1)\sin(\theta_2)\cos(\phi_1 - \phi_2) + \cos(\theta_1)\cos(\theta_2) \qquad (3.2)\end{aligned}$$

The above derivation makes explicit use of some of the properties listed in section 1. Equation 3.2 is known as the cosine rule and it is the basic formula in astronavigation.

A spherical triangle has as its sides segments of great circles, i.e. circles around the sphere's centre, as shown in figure 3. A, B and C are points on the surface of the sphere. The sides of the spherical triangle are the angles between the directions OB, OC, OA, OC and OA, OB. We denote them as a, b and c. and when we take the radius of the sphere as =1, they stand in radians. The angles on the surface of the sphere will be α, β and γ, chosen such that α is opposite to a, β opposite to b and γ to c.

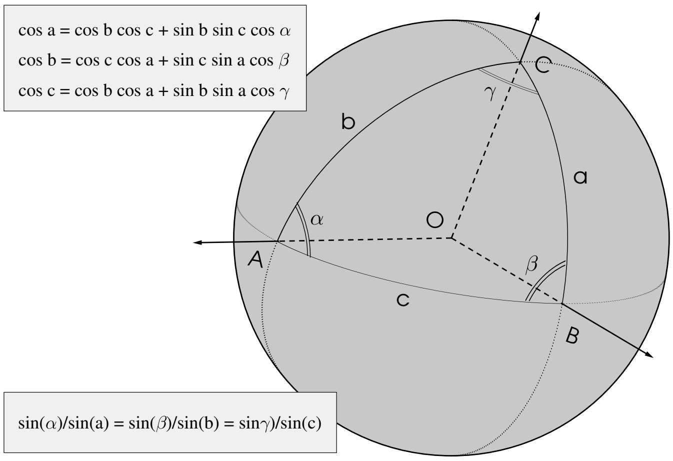

Figure 3: Spherical triangle. Cosine rule and sine rule.

The cosine rules, displayed in figure 3, are found by making the connection to figure 2, thus: Take as z-axis the direction along OC. The correspondence is then:

$$\begin{array}{rr} OA \Longleftrightarrow \hat{\mathbf{r}}_1 & OB \Longleftrightarrow \hat{\mathbf{r}}_2 \\ b \Longleftrightarrow \theta_1 & a \Longleftrightarrow \theta_2 \\ \gamma \Longleftrightarrow (\phi_1 - \phi_2) & c \Longleftrightarrow \Theta \end{array}$$

In the similar way, the z-axis may be taken along OA to find a or along OB for evaluating b. The resulting forms are known as of the cosine rule or, more formally:

Cosine rule of the first kind

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(\alpha) \tag{3.3}$$
$$\cos(b) = \cos(a)\cos(c) + \sin(a)\sin(c)\cos(\beta) \tag{3.4}$$
$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(\gamma) \tag{3.5}$$

Another useful formula is the *sine rule*:

$$\frac{\sin(\alpha)}{\sin(a)} = \frac{\sin(\beta)}{\sin(b)} = \frac{\sin(\gamma)}{\sin(c)} \tag{3.6}$$

Proof of the sine rule

From the cosine rule follows:

$$\cos(\alpha) = \frac{\cos(a) - \cos(b)\cos(c)}{\sin(b)\sin(c)}$$

which, after squaring and making use of the fact that $sin^2(\alpha) = 1 - cos^2(\alpha)$ and some elementary calculus, gives:

$$\frac{\sin^2(\alpha)}{\sin^2(a)} = \frac{1 - cos^2(a) - cos^2(b) - cos^2(c) + 2\cos(a)\cos(b)\cos(c)}{\sin^2(a)\sin^2(b)\sin^2(c)}$$

The right hand side is fully symmetric in a, b en c and is invariant for interchanging them:

$$\frac{\sin^2(\alpha)}{\sin^2(a)} = \frac{\sin^2(\beta)}{\sin^2(b)} = \frac{\sin^2(\gamma)}{\sin^2(c)} \tag{3.7}$$

Because $\sin(\alpha)$ and $\sin(a)$ are both positive or both negative, and likewise so for $\sin(\beta)$ and $\sin(b)$ and for $\sin(\gamma)$ and $\sin(c)$ we may leave off the squares, giving the sine rule as we know it.

4 More spherical trigonometric relations

In this section we give some more relations that are or have been in use.

Cosine formula of the second kind:

$$\cos(\alpha) = -\cos(\beta)\cos(\gamma) + \sin(\beta)\sin(\gamma)\cos(a) \tag{4.1}$$
$$\cos(\beta) = -\cos(\alpha)\cos(\gamma) + \sin(\alpha)\sin(\gamma)\cos(b) \tag{4.2}$$
$$\cos(\gamma) = -\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)\cos(c) \tag{4.3}$$

Relations for half angles:

These relations are of historical importance. While the cosine rule is a mixed form of multiplication and addition, forms that are strictly of product form were the preferred forms in the past. Their logarithms are sums of logarithms so that a practitioner could access them with the help of log tables without having to do multiplication. With the abbreviation $s = (a + b + c)/2$ the following relations for the half angles hold:

$$\cos^2(\tfrac{1}{2}\alpha) = \frac{\sin(s)\sin(s-a)}{\sin(b)\sin(c)} \tag{4.4}$$
$$\cos^2(\tfrac{1}{2}\beta) = \frac{\sin(s)\sin(s-b)}{\sin(a)\sin(c)} \tag{4.5}$$
$$\cos^2(\tfrac{1}{2}\gamma) = \frac{\sin(s)\sin(s-c)}{\sin(a)\sin(b)} \tag{4.6}$$

A variant is:

$$\sin^2(\tfrac{1}{2}\alpha) = \frac{\sin(s-b)\sin(s-c)}{\sin(b)\sin(c)} \quad (4.7)$$

$$\sin^2(\tfrac{1}{2}\beta) = \frac{\sin(s-a)\sin(s-c)}{\sin(a)\sin(c)} \quad (4.8)$$

$$\sin^2(\tfrac{1}{2}\gamma) = \frac{\sin(s-a)\sin(s-b)}{\sin(a)\sin(b)} \quad (4.9)$$

And yet another version:

$$\sin^2(\tfrac{1}{2}a) = \sin^2(\tfrac{1}{2}(b+c)) - \sin(b)\sin(c)\cos^2(\tfrac{1}{2}\alpha) \quad (4.10)$$

$$\sin^2(\tfrac{1}{2}b) = \sin^2(\tfrac{1}{2}(a+c)) - \sin(a)\sin(c)\cos^2(\tfrac{1}{2}\beta) \quad (4.11)$$

$$\sin^2(\tfrac{1}{2}c) = \sin^2(\tfrac{1}{2}(a+b)) - \sin(a)\sin(b)\cos^2(\tfrac{1}{2}\gamma) \quad (4.12)$$

The above formula's all come in sets of three as cyclic permutations of the argument pairs (a,α), (b,β) en (c,γ). Interchange of any of the three pairs does not give a new formula. They are invariant under both odd and even permutations. As an example the cosine formula $\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(\alpha)$ is symmetric in b en c, while β and γ are absent.

There are, however, other formula's that do not have this symmetry and where an odd permutation does yield a new formula. Such relations come in sets of six, as is the case for the following examples.

The sine-cosine formula of the first kind:

$$\sin(a)\cos(\beta) = \cos(b)\sin(c) - \sin(b)\cos(c)\cos(\alpha) \quad (4.13)$$

$$\sin(a)\cos(\gamma) = \cos(c)\sin(b) - \sin(c)\cos(b)\cos(\alpha) \quad (4.14)$$

$$\sin(b)\cos(\gamma) = \cos(c)\sin(a) - \sin(c)\cos(a)\cos(\beta) \quad (4.15)$$

$$\sin(b)\cos(\alpha) = \cos(a)\sin(c) - \sin(a)\cos(c)\cos(\beta) \quad (4.16)$$

$$\sin(c)\cos(\alpha) = \cos(a)\sin(b) - \sin(a)\cos(b)\cos(\gamma) \quad (4.17)$$

$$\sin(c)\cos(\beta) = \cos(b)\sin(a) - \sin(b)\cos(a)\cos(\gamma) \quad (4.18)$$

The sine-cosine formula of the second kind:

$$\sin(\alpha)\cos(b) = \cos(\beta)\sin(\gamma) + \sin(\beta)\cos(\gamma)\cos(a) \quad (4.19)$$

$$\sin(\alpha)\cos(c) = \cos(\gamma)\sin(\beta) + \sin(\gamma)\cos(\beta)\cos(a) \quad (4.20)$$

$$\sin(\beta)\cos(c) = \cos(\gamma)\sin(\alpha) + \sin(\gamma)\cos(\alpha)\cos(b) \quad (4.21)$$

$$\sin(\beta)\cos(a) = \cos(\alpha)\sin(\gamma) + \sin(\alpha)\cos(\gamma)\cos(b) \quad (4.22)$$

$$\sin(\gamma)\cos(a) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)\cos(c) \quad (4.23)$$

$$\sin(\gamma)\cos(b) = \cos(\beta)\sin(\alpha) + \sin(\beta)\cos(\alpha)\cos(c) \quad (4.24)$$

5 Rectangular and quadrantal spherical triangles. Napier's rules

Consider a spherical triangle such as in figure 3. When one of the angles α, β or γ equals 90°, the triangle is called rectangular. It is called quandrantal, if one of the sides, a, b or c equals 90°, since it spans just a quadrant as viewed from the center of the sphere.
For example, let α=90°. Then, since $\cos(\alpha) = 0$ and $\sin(\alpha) = 1$, equations that contain α simplify. The sine or cosine of each of the other five elements a, b, c, β en γ, may be expressed as a product tangents or cotangents of its adjacent elements or, alternatively, as a product of sines or cosines of its opposite elements.

Relations for $\alpha = 90°$

$$\begin{aligned}
\cos(a) &= \cot(\beta)\cot(\gamma) && (5.1)\\
&= \cos(b)\cos(c) && (5.2)\\
\sin(b) &= \tan(c)\cot(\gamma) && (5.3)\\
&= \sin(a)\sin(\beta) && (5.4)\\
\sin(c) &= \tan(b)\cot(\beta) && (5.5)\\
&= \sin(a)\sin(\gamma) && (5.6)\\
\cos(\beta) &= \cot(a)\tan(c) && (5.7)\\
&= \cos(b)\sin(\gamma) && (5.8)\\
\cos(\gamma) &= \cot(a)\tan(b) && (5.9)\\
&= \sin(\beta)\cos(c) && (5.10)
\end{aligned}$$

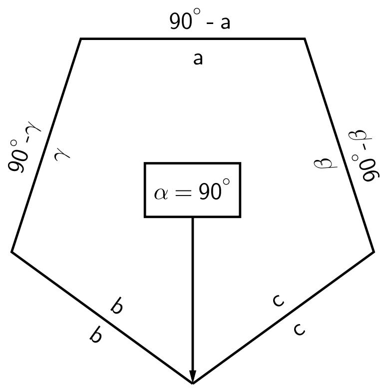

Figure 4: Napier's rules for $\alpha = 90°$

As an example of how the above equations may be derived: using $\cos(\alpha) = 0$ in eq. 4.1, the cosine formula of the second kind, produces eq. 5.1. Similarly, eq. 5.2 follows from eq. 3.3, the cosine formula of the first kind. In very much the same way we may derive all of the above equations 5.x.
A way to obtain these simplified formulas by a common recipe has been formulated by John Napier (1550–1617). The procedure, which is really a mnemonic, is illustrated in figure 4. Starting from the 90° element, which is α in this case, the other five elements c, β, a, γ and b are associated with the five sides of a pentagon in anti-clockwise order. On the outside of each side their arguments are written as they will be used in Napier's rules. These arguments are just the element itself for the neighbouring sides b and c and their complements for the other three, β, a and γ. This done, each of the five sides, when chosen as "the middle" has two adjacent sides and two opposite sides. The following table lists this correspondence for their associated arguments for the case $\alpha = 90°$.

middle	adjacent 1	adjacent 2	opposite 1	opposite 2
$90° - a$	$90° - \beta$	$90° - \gamma$	c	b
b	c	$90° - \gamma$	$90° - a$	$90° - \gamma$
c	b	$90° - \beta$	$90° - \gamma$	$90° - a$
$90° - \beta$	$90° - a$	c	b	$90° - \gamma$
$90° - \gamma$	$90° - a$	b	$90° - \beta$	c

Napier's rules are formulated in terms of these arguments:

sin(middle) = product of the tangents of the adjacents.
sin(middle) = product of the cosines of the opposites.

As an example, eq. 5.1 is produced via Napier's rules thus:
$\sin(90° - a) = \boxed{\cos(a)} = \tan(90° - \beta)\tan(90° - \beta) = \boxed{\cot(\beta)\cot(\gamma)}$
Eq. 5.2 is found via $\sin(90° - a) = \boxed{\cos(a)} = \boxed{\cos(b)\cos(c)}$

Relations for a = 90°

$$\begin{aligned}
\cos(\alpha) &= -\cot(b)\cot(c) && (5.11)\\
&= -\cos(\beta)\cos(\gamma) && (5.12)\\
\sin(\beta) &= \tan(\gamma)\cot(c) && (5.13)\\
&= \sin(\alpha)\sin(b) && (5.14)\\
\sin(\gamma) &= \tan(\beta)\cot(b) && (5.15)\\
&= \sin(\alpha)\sin(c) && (5.16)\\
\cos(b) &= -\cot(\alpha)\tan(\gamma) && (5.17)\\
&= \cos(\beta)\sin(c && (5.18)\\
\cos(c) &= -\cot(\alpha)\tan(\beta) && (5.19)\\
&= \sin(b)\cos(\gamma) && (5.20)
\end{aligned}$$

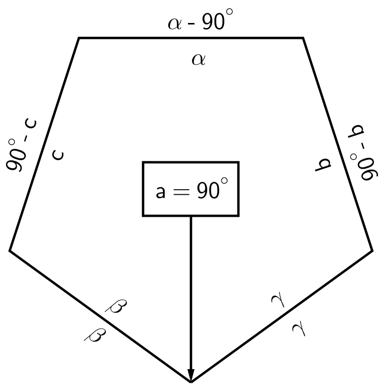

Figure 5: Napier's rules for a = 90°

The relations and figure 5, above, are given for the case that the side $a = 90°$. **Napier's rules hold just the same when the right-angled element is a side, i.e. a quadrantal angle. There is one important difference: the argument of the angle opposite to it, is again its complement, but with an additional minus sign.**

As shown in figure 5, the argument, that is associated with the angle α is $(\alpha - 90°)$ in this case. This introduces a minus sign in eqs, 5.11, 5.12, 7.17 and 5.19, since both the sine and the tangent change sign upon changing the sign of their argument, while the cosine is even.

Tables and figures for $\beta = 90°$ and for $b = 90°$ are simply obtained by making the cyclic permutation $\alpha, c, \beta, a, \gamma, b \Rightarrow \beta, a, \gamma, b, \alpha, c$ and those for $\gamma = 90°$ and for $c = 90°$ by making the further permutation $\alpha, c, \beta, a, \gamma, b \Rightarrow \gamma, b, \alpha, c, \beta, a$.

Although Napier's rules are meant to make things easier, yet the practitioner must be very careful in applying them. Identifying the correct argument for each of the elements is a potential source of mistake. Often the more secure way will be to proceed by finding under section 3 or 4 an appropriate formula in which the 90° element appears in a cosine and thus vanishes.

Tables and figures for the remaining four cases with respectively β, b, γ, c as the 90° element, are given below without comments.

Relations for $\beta = 90°$

$$\begin{aligned}
\cos(b) &= \cot(\gamma)\cot(\alpha) && (5.21)\\
&= \cos(c)\cos(a) && (5.22)\\
\sin(c) &= \tan(a)\cot(\alpha) && (5.23)\\
&= \sin(b)\sin(\gamma) && (5.24)\\
\sin(a) &= \tan(c)\cot(\gamma) && (5.25)\\
&= \sin(b)\sin(\alpha) && (5.26)\\
\cos(\gamma) &= \cot(b)\tan(a) && (5.27)\\
&= \cos(c)\sin(\alpha) && (5.28)\\
\cos(\alpha) &= \cot(b)\tan(c) && (5.29)\\
&= \sin(\gamma)\cos(a) && (5.30)
\end{aligned}$$

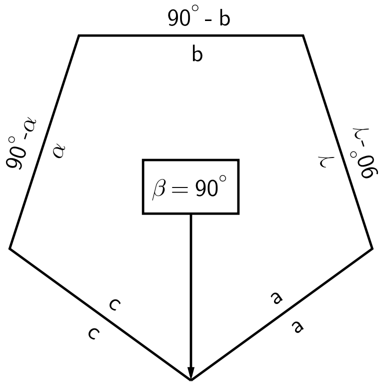

Figure 6: Napier's prodedure for $\beta = 90°$

Relations for b = 90°

$$\begin{aligned}
\cos(\beta) &= -\cot(c)\cot(a) && (5.31)\\
&= -\cos(\gamma)\cos(\alpha) && (5.32)\\
\sin(\gamma) &= \tan(\alpha)\cot(a) && (5.33)\\
&= \sin(\beta)\sin(c) && (5.34)\\
\sin(\alpha) &= \tan(\gamma)\cot(c) && (5.35)\\
&= \sin(\beta)\sin(a) && (5.36)\\
\cos(c) &= -\cot(\beta)\tan(\alpha) && (5.37)\\
&= \cos(\gamma)\sin(a) && (5.38)\\
\cos(a) &= -\cot(\beta)\tan(\gamma) && (5.39)\\
&= \sin(c)\cos(\alpha) && (5.40)
\end{aligned}$$

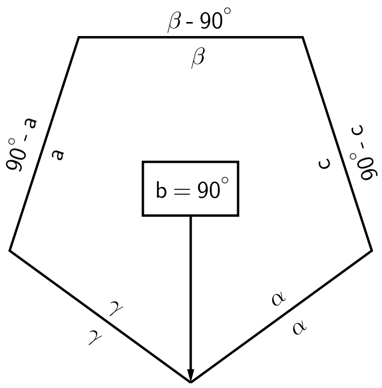

Figure 7: Napier's rules for b = 90°

Relations for $\gamma = 90°$

$$\begin{aligned}
\cos(c) &= \cot(\alpha)\cot(\beta) && (5.41)\\
&= \cos(a)\cos(b) && (5.42)\\
\sin(a) &= \tan(b)\cot(\beta) && (5.43)\\
&= \sin(c)\sin(\alpha) && (5.44)\\
\sin(b) &= \tan(a)\cot(\alpha) && (5.45)\\
&= \sin(c)\sin(\beta) && (5.46)\\
\cos(\alpha) &= \cot(c)\tan(b) && (5.47)\\
&= \cos(a)\sin(\beta) && (5.48)\\
\cos(\beta) &= \cot(c)\tan(a) && (5.49)\\
&= \sin(\alpha)\cos(b) && (5.50)
\end{aligned}$$

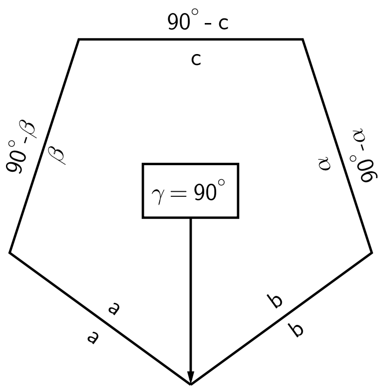

Figure 8: Napier's rules for $\gamma = 90°$

Relations for c = 90°

$$
\begin{aligned}
\cos(\gamma) &= -\cot(a)\cot(b) && (5.51)\\
&= -\cos(\alpha)\cos(\beta) && (5.52)\\
\sin(\alpha) &= \tan(\beta)\cot(b) && (5.53)\\
&= \sin(\gamma)\sin(a) && (5.54)\\
\sin(\beta) &= \tan(\alpha)\cot(a) && (5.55)\\
&= \sin(\gamma)\sin(b) && (5.56)\\
\cos(a) &= -\cot(\gamma)\tan(\beta) && (5.57)\\
&= \cos(\alpha)\sin(b) && (5.58)\\
\cos(b) &= -\cot(\gamma)\tan(\alpha) && (5.59)\\
&= \sin(a)\cos(\beta) && (5.60)
\end{aligned}
$$

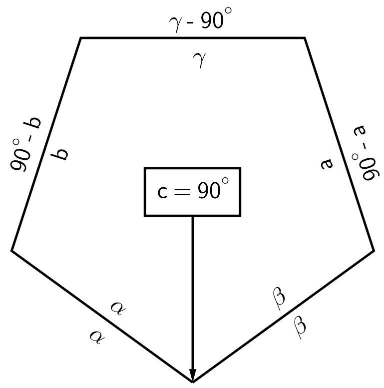

Figure 9: Napier's rules for c = 90°

This compendium is a combination of notes that were handed out for courses on Vibrations and Waves and on the History of Navigation and Astronavigation, both given at the University of Groningen.

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